Mechanical vibration

Vibration is defined as a motion which repeats after equal interval of time and is also a periodic motion. Vibration is also termed as oscillation.

Mechanical vibration is the study of of oscillatory motion of dynamic system.

The swinging of a pendulum and the motion of a plucked string are typical example of vibration.

Cause of vibration

- Unbalanced forces in M/C.
- Dry friction between two mating surface.
- External excitation.
- Earth quake
- wind
- misalignment.

Harmful effect

- Excessive stresses in M/C parts.
- Undesirable noise
- Looseness of parts and partial or complete failure of parts.

Uses of vibration

- vibrating conveyor
- vibrating screen
- Shaker
- stress relieving
- music instrument.

Elimination of vibration

- By removing the cause of vibration.
- vibration Isolation.
- Using shock absorber
- Installing dynamic vibration absorber.

Classification of vibration

Mechanical vibration

- Linear
  - Lumped
    - Single Degree of freedom (SDOF)
    - Multi Degree of freedom (MDOF)
      - Free
      - Forced
        - Damped
        - Undamped
          - Longitudinal
          - Transverse
          - Torsional
  - Continuous
    - Longitudinal
    - Lateral
    - Transverse
- Non linear
  - Free
  - Forced
  - Transient
  - Random

Linear vibration :- If all the basic element of vibrating system ie mass, spring and damper behaves linearly called linear vibration. The differential equation of the system are linear.

Non linear vibration :-

If any of the basic components of a vibrating system behaves non-linearly, the resulting vibration is known as Non-linear vibration.

→ the differential equations that governed non linear vibrating system are non-linear.

→ In non linear the principle of super position does not hold ~~good~~ and technique of analysis is well known

Lumped vibration - It is also called as discrete vibration system.

→ In lumped vibration the natural frequency and mode shapes are equal to degree of freedom.

→ It represents in ordinary differential equation.

→ It posses only one independent displacement.

continuous vibration

→ continuous vibration system represents in partial differential equation

→ It contains indefinite number of possible independent displacements.

Free vibration - If a system after initial disturbance is left to vibrate on its own, the ensuring vibration is known as free vibration.

ex- simple pendulum.

Forced vibration - If a system subjected to an external force, the resulting vibration is known as forced vibration.

ex- Diesel engine.

Damped vibration - If any energy is lost/ dissipated during oscillation, such vibration are known as undamped vibration.

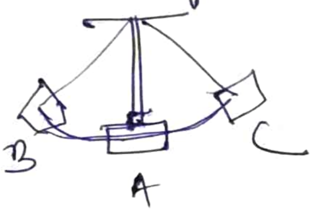
→ In this vibration the amplitude of the vibration reduced in every cycle.

Un-damped vibration - If the energy is lost or dissipated during oscillations, such vibration is known as undamped vibration.

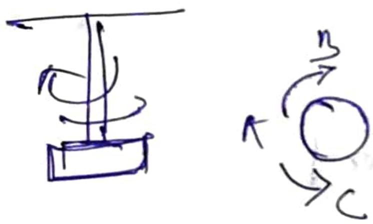
→ In this vibration there is very little influence of natural frequency.

Longitudinal vibration :- If the mass of the shaft moves parallel to the axis of shaft, then the vibration is known as longitudinal vibration.

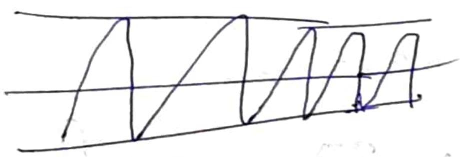
Transverse vibration → If the mass of the shaft moves vibrates perpendicular to the shaft, the vibration are known as trans vibration.



Torsional vibration → If the shaft gets alternatively twisted and untwisted on account of an alternate torque on the disc, is said to execute torsional vibration.



Transient vibration – It is the temporarily sustained vibration where the amplitude of the vibrating system decreases continuously.



Random vibration – If the magnitude of excitation acting on the vibratory system at a given time cannot be predicted, the resulting vibration is known as random vibration or non-deterministic vibration.

Terminology

Period of vibration (Time period) - It is the time interval after the motion is repeated itself.

→ It represents in second.

cycle - It is the motion completed during one time period.

Frequency - It is the no. of cycles described in one second

It is denoted by (f).

Unit = Hz = cycles/second

Amplitude - It is the maximum displacement of a vibrating body from the mean position.

Circular frequency - It represents in radian per second.

It is denoted by $\omega = \frac{2\pi}{T} = 2\pi f$ = rad/sec

cyclic frequency - It is the ratio of no. of cycles per second.

$$\left(f = \frac{1}{T}\right)$$

Natural frequency: (ω_n) - It is the frequency of free vibration of the system.

Unit = rad/sec.

phase difference:- It is the angle between two rotating vectors executing simple harmonic motion of same frequency.

ex - If the 1st vector $x_1 = X \sin \omega t$
The second vector $x_2 = X \sin(\omega t + \phi)$

where ϕ is the phase difference between x_1 and x_2.

Simple Harmonic Motion (SHM):- (Motion with constant angular velocity)
A vibration with acceleration proportional to displacement and directed towards the mean position is known as (SHM), Simple harmonic motion.

. Periodic - - A motion that repeats itself after equal interval of time

Degree of freedom:- ~~the~~ (DOF) :- The minimum number of indipendent coordinatis required to describe the motion of the mechanical system at any instant of time defines degree of freedom of the system.

Note.
→ A free particle undergoing general motion in space will have three degree of freedom.

→ A rigid body will have 6 (six) degree of freedom. (three component of position and three angle defining the position.

→ A continious elastic body will require an infinite number of co-ordinates.

Springs are in parallel and series

spring element

Considering the linear spring of negligible mass and damping, A force is developed in the spring when ever there is relative motion between the two ends of spring.

The spring force (F) is proportional to the deformation is given by

$F \propto x$

Spring force:- $\boxed{F = kx}$

where k is the spring stiffness or spring constant.

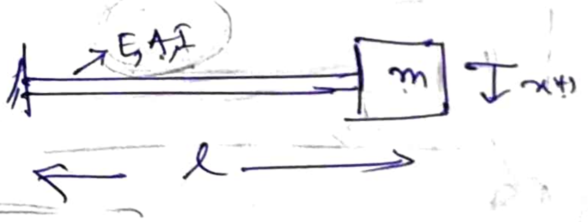
x = deformation.

The work done 'U' in deforming a spring is stored as strain or potential energy in the spring is given by

$\boxed{U = \frac{1}{2} k x^2}$ → spring potential energy or work done.

* Calculate the ~~spring~~ stiffness of the cantilever beam with an end mass ~~mass~~ 'm' as shown.

Soln



As it is an elastic beam, it also behaves like a spring.

As per the strength of material the static deflection δ of cantilever beam is

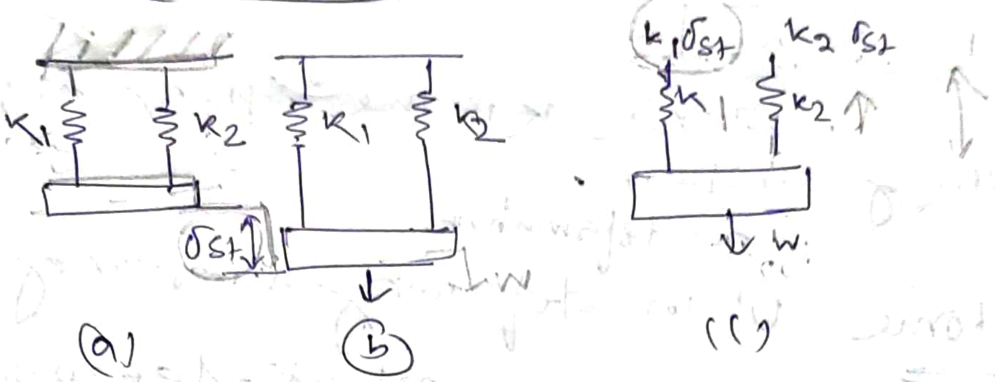
$$\delta_{st} = \frac{Wl^3}{3EI}, \quad (W = mg)$$

As $F = kx = k\delta_{st} = W$

$$\cancel{W} = k \frac{\cancel{W}l^3}{3EI}$$

$$\boxed{k = \frac{3EI}{l^3}}$$

Spring are in parallel



when the load applied, the system undergoes a static deflection δ_{st}. For FBD

$$W = K_1 \delta_{st} + k_2 \delta_{st} \quad \text{--- (1)}$$

If k_{eq} denotes the equivalent spring constant of combination of two keys, then for the same static deflection

$$W = k_{eq} \delta_{st} \quad \text{--- (2)}$$

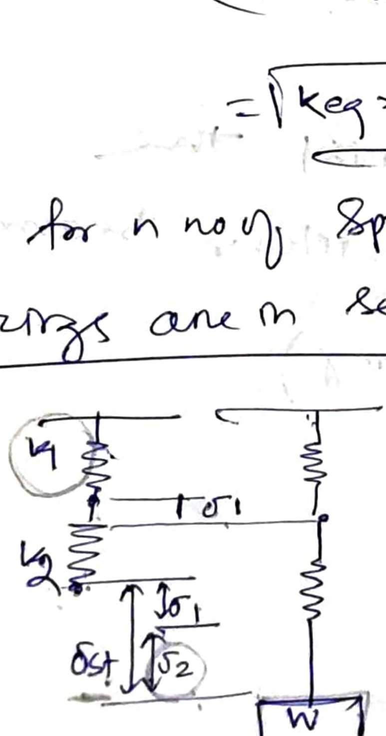
equating (1) & (2)

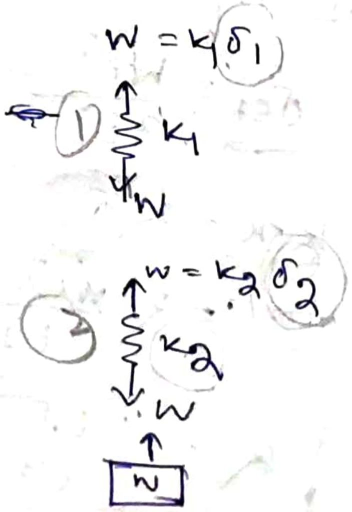
$$(K_1 + k_2) \cancel{\delta_{st}} = k_{eq} \cancel{\delta_{st}}$$

$$\Rightarrow \boxed{k_{eq} = k_1 + k_2}$$

for n no of spring $\boxed{k_{eq} = k_1 + k_2 + \dots + k_n}$

Springs are in series :-





Under the action of load 'W', spring (1) & (2) under goes elongation δ_1 & δ_2 respectively.
The Total elongation.

$$\delta_{st} = \delta_1 + \delta_2 \quad \text{——} (1)$$

Since both springs are subjected to same force W, then for equilibrium.

$$\left.\begin{aligned} W &= k_1 \delta_1 \\ W &= k_2 \delta_2 \end{aligned}\right\} \quad \text{——} (2)$$

If keq denotes the equivalent spring constant, then for same static deflection

$$W = K_{eq}\,\delta_{st} \quad \text{——} (3)$$

form eq (2) & (3).

$$k_1 \delta_1 = k_2 \delta_2 = k_{eq}\,\delta_{st}$$

$$\text{or } \delta_1 = \frac{k_{eq}\,\delta_{st}}{k_1} \qquad \delta_2 = \frac{k_{eq}\,\delta_{st}}{k_2} \quad \text{——} (4)$$

substituting the eqn (4) in (1).

$$\frac{K_{eq}\,\cancel{\delta_{st}}}{k_1} + \frac{k_{eq}\,\cancel{\delta_{st}}}{k_2} = \cancel{k_{eq}}\,\cancel{\delta_{st}}$$

$$\boxed{\text{or } \frac{1}{k_1} + \frac{1}{k_2} = \frac{1}{k_{eq}}} \quad \text{——} (5)$$

Generalised to equation (5) for 'n' members of springs in series

$$\boxed{\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \cdots + \frac{1}{k_n}}$$

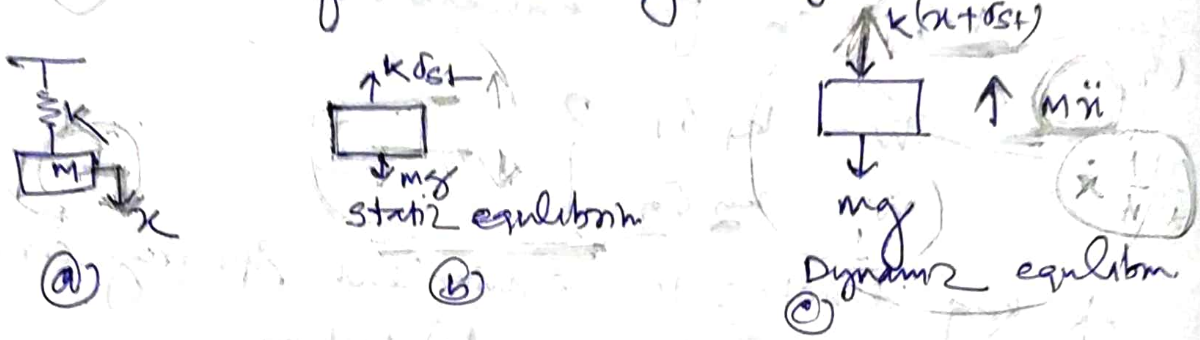
Undamped free vibration

Considering the spring-mass system as shown,

~~Let the stiffness of the spring be k~~

Let k be the stiffness of the spring in N/mm or kN/m

m = mass of the spring, in kg.



From static equilibrium fig (b). equation:

$$\boxed{mg = k\delta_{st}} \quad \text{——— (1)}$$, where δ_{st} = static deflection in mm.

The forces acting on mass M, when the systems displaced downward by the amount x.

$M\ddot{x}$ = Inertia force upward.

mg = weight downward.

kx = spring force upward (Resisting force)

From dynamic equilibrium fig-(c)

$$\uparrow \boxed{M\ddot{x} + k(x+\delta_{st}) = mg} \downarrow \text{——— (2)}$$

$$m\ddot{x} + kx + \cancel{k\delta_{st}} = \cancel{mg}$$

$$\boxed{M\ddot{x} + kx = 0}$$, (as $mg = k\delta_{st}$) eqn-(1) ——— (2)

eqn (3) is known as equation of motion of the spring-mass system of vibration.

from eq^n (3)

$\ddot{x} + K/m = 0$, $\boxed{\ddot{x} + \omega_n^2 x = 0}$ — (4)

let $\omega_n^2 = K/m$,

then $\boxed{\omega_n = \sqrt{K/m} = \text{rad/sec}}$ — (5)

where ω_n = natural frequency of the system.

Natural frequency = $\boxed{f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{K/m}}$ = Hz

Time period = $\boxed{T = \frac{2\pi}{\omega_n} = 2\pi\sqrt{M/K}}$ = sec.

Note: Also As $\boxed{\omega_n = \sqrt{K/m} = \sqrt{\frac{Kg}{mg}} = \sqrt{\frac{\cancel{K}g}{\cancel{K}\delta_{st}}} = \sqrt{\frac{g}{\delta_{st}}}}$

Solution to equation $\ddot{x} + \omega_n^2 x = 0$

$\ddot{x} + \omega_n^2 x = 0$

The equation can be written as $(D^2 + \omega_n^2)\, x = 0$ — (1)

where $D^2 = \frac{d^2}{dt^2}$

Auxillary equation

$D^2 + \omega_n^2 = 0$

$D = \pm i\omega_n$

Soln of eq^n (1)

$\boxed{x(t) = A \sin \omega_n t + B \cos \omega_n t}$

where A & B are constants, can be found out by BC's, i.e. $\left.\begin{array}{l} x(t) = x(0) \\ \dot{x}(t) = v(0) \end{array}\right]$ at $t=0$

Using BC's (i) ; $\boxed{B = x_0}$

BC's (ii) $\dot{x}(t) = \omega_n (A \cos \omega_n t - B \sin \omega_n t)$

$v_0 = A\omega_n$, $\boxed{A = \frac{v_0}{\omega_n}}$

Substituting the value of A & B, the solution of the equation becomes.

$$\boxed{x(t) = \frac{v_0}{\omega_n} \sin \omega_n t + x_0 \cos \omega_n t}$$

Also let $\frac{v_0}{\omega_n} = X \cos \phi$, $x_0 = X \sin \phi$, substituting in above

$$x(t) = X \sin(\omega_n t + \phi)$$

where $X = \sqrt{\left(\frac{v_0}{\omega_n}\right)^2 + x_0^2}$ is the Amplitude

phase angle $\phi = \tan^{-1}\left[\frac{x_0 \omega_n}{v_0}\right]$

Energy method

In a conservative ~~sys~~ system the sum of total energy is constant. In a vibratory system the energy is partly potential and partially kinetic.

According to law of conservation, we know that

$$KE + PE = constant \quad \text{——} (1)$$

differentiating the above eqn w.r.t time will be zero.

$$\frac{d}{dt}(KE + PE) = 0 \quad \text{——} (2)$$

$$\frac{d}{dt}\left(\frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2\right) = 0$$

$$\frac{1}{2}m\,\ddot{x}\dot{x} + \frac{1}{2}k\,x\cdot\dot{x} = 0$$

$$\boxed{m\ddot{x} + kx = 0} \quad \text{——} (3)$$

Releleigh's method

Assume the motion to be S.H.M, then

$$x = A\sin w_n t \quad \text{——} (1)$$

where: x = displacement of the body from mean position after time

A = max^m displacement from mean posi^n to extreme position.

differentiating w.r.t time,

$$\dot{x} = w_n A\cos w_n t.$$

max^m velocity at mean position, $\dot{x} = w_n A$, $(\cos w_n t = 1)$

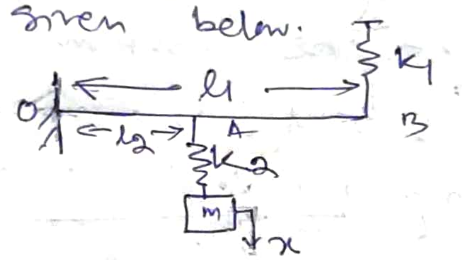
max^m KE at mean position $= \frac{1}{2}m\dot{x}^2 = \frac{1}{2}m w_n^2 A^2$ —— (2)

max^m P.E. at extreme position $= \frac{1}{2}kx^2$ —— (3)

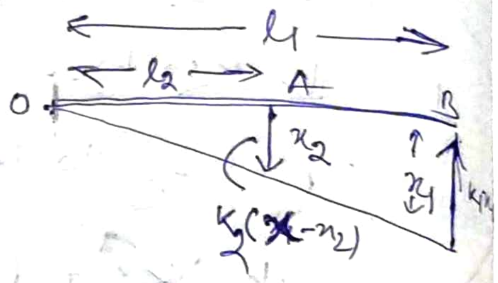
equating eq^n (2) & (3), we get

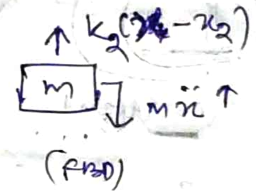
$$\frac{1}{2}m w_n^2 A^2 = \frac{1}{2}kA^2, \quad \boxed{w_n^2 = k/m}, \quad \boxed{w_n = \sqrt{k/m}}$$

Q-1 Find out the natural frequency of the system given below.



Sol^n - Draw the displacement diagram.





According to FBD

$$\boxed{m\ddot{x} + k_2 x - k_2 x_2 = 0} \quad \text{---} (1)$$

From the above similar triangle.

$$\frac{x_1}{x_2} = \frac{l_1}{l_2} \quad \text{---} (2)$$

Taking moment about 'O'.

$$k_1 x_1 l_1 - k_2 (x - x_2) l_2 = 0$$

$$= k_1 x_1 l_1 - k_2 x l_2 + k_2 x_2 l_2 = 0$$

$$= k_1 \frac{l_1^2}{l_2} x_2 - k_2 x l_2 + k_2 l_2 x_2 = 0$$

$$x_2 \left[\frac{k_2 l_2^2 + k_1 l_1^2}{l_2} \right] = k_2 x l_2$$

$$x_2 = \left[\frac{k_2 l_2^2 x}{k_2 l_2^2 + k_1 l_1^2} \right]$$

Substituting in eq^n (1), the equation becomes

$$m\ddot{x} + \left[\frac{k_2^2 l_2^2 + k_1 k_2 l_2^2 - k_2^2 l_2^2}{k_2 l_2^2 + k_1 l_1^2} \right] x = 0 \quad \text{---} (3)$$

Keq

$$\boxed{m\ddot{x} + k_{eq} x = 0}$$

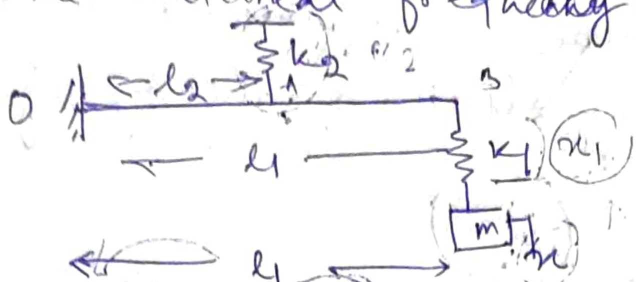
$$\ddot{x} + \frac{k_{eq}}{m} x$$

ω^2

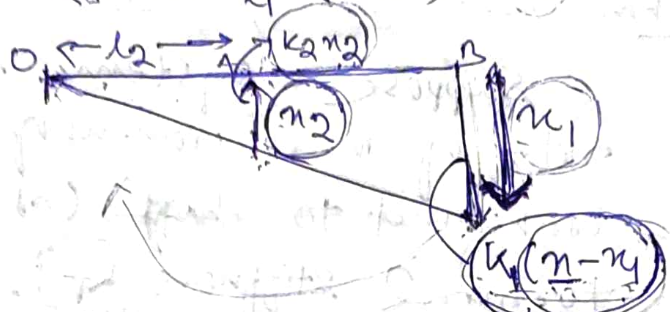
$$\boxed{\omega_n = \sqrt{\frac{k_1 k_2 l_1^2}{m (k_2 l_2^2 + k_1 l_1^2)}}} \text{ rad/sec.}$$

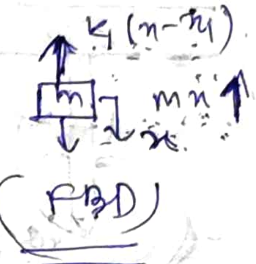
$$\boxed{k_{eq} = \frac{k_1 k_2 l_1^2}{k_2 l_2^2 + k_1 l_1^2}}$$

(a) Find out the natural frequency of the system:



Sol^n





According to FBD, using D-Alembert's principle,

$$\boxed{m\ddot{x} + k_1(x-x_1) = 0} \quad \text{--- (1)}$$

From above similar triangle,

$$\frac{x_1}{x_2} = \frac{l_1}{l_2} \quad \text{--- (2)}$$

Taking moment about 'O'.

$$\boxed{k_2 x_2 l_2 - k_1(x-x_1)\, l_1 = 0} \quad \text{--- (3)}$$

Substituting the value of x_2 from eq^n (2)

$$k_2 l_2 \left(\frac{l_2}{l_1}\right) x_1 - k_1 l_1 x + k_1 x_1 l_1 = 0$$

$$k_2 \frac{l_2^2}{l_1} x_1 + k_1 x_1 l_1 = l_1 k_1 x$$

$$\frac{(k_2 l_2^2 + k_1 l_1^2)\, x_1}{k_1 l_1^2} = x \quad , \text{ or } \quad \boxed{x_1 = x\left(\frac{k_1 l_1^2}{k_2 l^2 + k_1 l_1^2}\right)}$$

Substituting the value of x_1 in eq^n (1)

$$m\ddot{x} + \left[\frac{k_1 k_2 l_2^2 + \cancel{k_1 l_1^2} - \cancel{k_1 l_1^2}}{k_1 l_1^2 + k_2 l_2^2}\right] x = 0$$

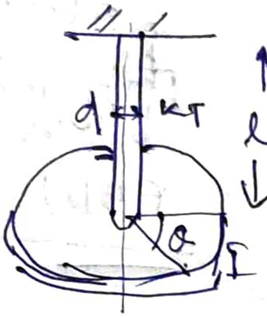
or $$m\ddot{x} + \frac{k_1 k_2 l_2^2}{(k_2 l_2^2 + k_1 l_1^2)}\, x = 0 \quad \left[\because M\ddot{x} + k_{eq} x = 0\right]$$

$$\boxed{k_{eq} = \frac{k_1 k_2 l_2^2}{k_1 l_1^2 + k_2 l_2^2}}$$

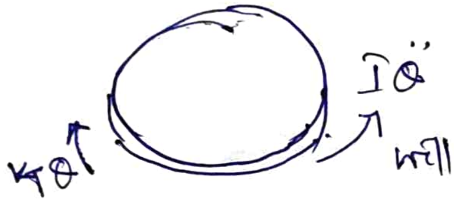
$$\boxed{\omega_n = \sqrt{\frac{k_1 k_2 l_2^2}{m(k_2 l_2^2 + k_1 l_1^2)}}}$$

MOTION N

Torsional vibration



Suppose a system having a rotor of mass moment of inertia 'I' connected to shaft (at its end) of torsional stiffness k_T, is twisted by an angle θ.



(FBD)

The body is rotated through angle θ and released, the torsional vibration will result.

Then the equation of motion is

$$I\ddot{\theta} + k_T\theta = 0 , \quad (\text{where } k_T\theta = \text{Restoring torque})$$

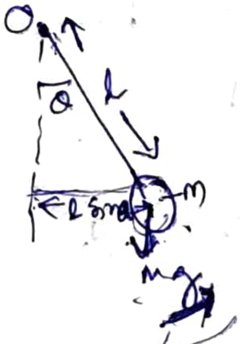
or $$\ddot{\theta} + \frac{k_T}{I}\theta = 0 \quad \text{—— (1)}$$

Putting $\omega_n^2 = \frac{k_T}{I}$, the equation (1) will become

$$\boxed{\ddot{\theta} + \omega_n^2\theta = 0}$$

$m\ddot{x} + kx = 0$

Simple pendulum



If we give small angular displacement 'θ' in anticlockwise direction, then the equation of motion,

$$I\ddot{\theta} + mgl\sin\theta = 0$$ (for very small value of θ, so $\sin\theta \approx \theta$)

$$I\ddot{\theta} + mgl\theta = 0$$

$$\ddot{\theta} + \frac{mgl}{ml^2}\theta = 0$$

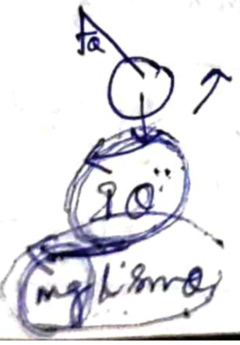
I = mass MI of bob = ml^2

$$\ddot{\theta} + \frac{g}{l}\theta = 0$$

$$\boxed{\omega_n = \sqrt{g/l}}$$

$$\boxed{f_n = \frac{1}{2\pi}\sqrt{g/l}}$$

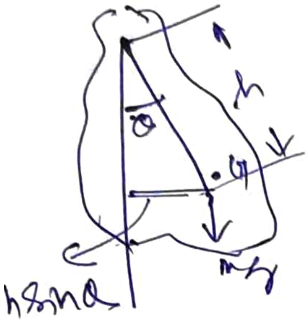
FBD



Compound pendulum

An rigid body having angular oscillation about a point of suspension is called a compound pendulum.

The equation of motion



$$I\ddot{\theta} + mgh\sin\theta = 0$$

$$(mk^2 + mh^2)\ddot{\theta} + mgh\,\theta = 0 \quad (\text{as } \theta = \text{to be very small})$$

$(k = \text{radius of gyration})$

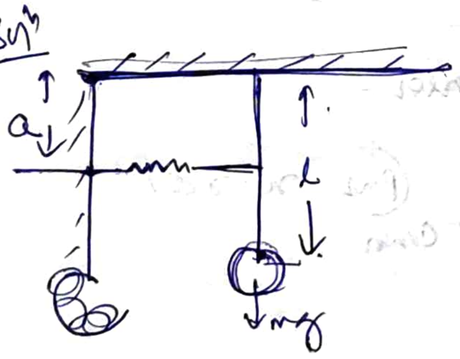
$$\ddot{\theta} + \frac{mgh}{mk^2 + mh^2}\theta = 0$$

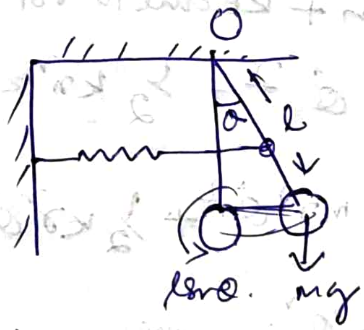
$$\ddot{\theta} + \frac{gh}{k^2 + h^2}\theta = 0$$

$$\omega_n = \sqrt{\frac{gh}{k^2 + h^2}} \text{ rad/sec.}$$

Q. find out the natural frequency of the system given below. The spring is in unstretched position when the pendulum rod is vertical.

Sol^n





(When given angular displacement θ to right)

Eq^n of motion, $I\ddot{\theta} + mgl\sin\theta + (ka\sin\theta)(a\cos\theta) = 0$

$$\cancel{I\ddot{\theta} + mgl\sin\theta + ka(a\theta)} = 0$$

$$I\ddot{\theta} + mgl\sin\theta + ka \cdot a\theta = 0, \quad \left(\begin{array}{l}\text{Put } I = ml^2 \\ \sin\theta \simeq \theta \\ \text{ } a\theta \\ (\cos\theta = 1)\end{array}\right)$$

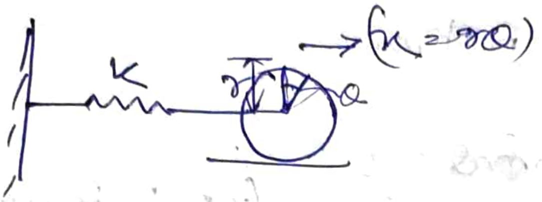
$$ml^2\ddot{\theta} + (mgl + ka^2)\theta = 0$$

$$\ddot{\theta} + \left(\frac{mgl + ka^2}{ml^2}\right)\theta = 0$$

$$\text{or } \omega_n = \sqrt{\frac{mgl + ka^2}{ml^2}}$$

Q. A circular cylinder connected by spring. It is free to roll on horizontal rough surface without slipping determine the natural frequency.

Sol^n



Using D-Alembert's principle.

$$M\ddot{x}\cdot r + I\ddot{\theta} + (kx)\cdot r = 0$$

$$M(r\ddot{\theta})\cdot r + \frac{1}{2}Mr^2\ddot{\theta} + k(r\theta)\cdot r = 0 \qquad \left[\text{put } I = \frac{1}{2}Mr^2,\ x = r\theta,\ \ddot{x} = r\ddot{\theta}\right.$$

$$Mr^2\ddot{\theta} + \frac{1}{2}Mr^2\ddot{\theta} + kr^2\theta = 0$$

$$\left(m + \frac{1}{2}m\right)\ddot{\theta} + k\theta = 0$$

$$\ddot{\theta} + \frac{2k}{3m}\theta = 0$$

$$\left(\omega_n = \sqrt{\frac{2k}{3m}}\ \text{rad/sec}\right)$$

Using energy method

KE due to translation + KE due to rot^n + PE due to spring = const

$$= \frac{1}{2}m\dot{x}^2 + \frac{1}{2}I\dot{\theta}^2 + \frac{1}{2}kx^2 = \text{const.}$$

$$= \frac{1}{2}mr^2\dot{\theta}^2 + \frac{1}{2}\cdot\frac{1}{2}mr^2\dot{\theta}^2 + \frac{1}{2}kr^2\theta^2 = \text{const.} \quad (\text{put } x = r\theta)$$

$$= \frac{3}{4}mr^2\dot{\theta}^2 + \frac{1}{2}kr^2\theta^2 = \text{const.}$$

Differentiating above equation w.r.t time.

$$\frac{3}{\cancel{4}_{2}}mr^2\cdot\cancel{2}\cancel{\dot{\theta}}\ddot{\theta} + \frac{1}{\cancel{2}}\cancel{2}kr^2\cancel{\dot{\theta}}\theta = 0$$

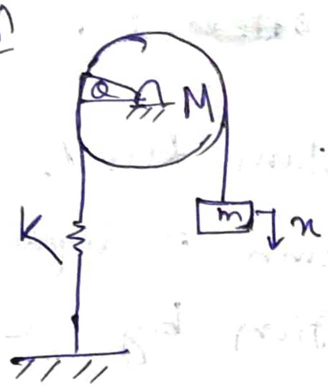
$$\frac{3}{2}mr^2\ddot{\theta} + kr^2\theta = 0$$

$$\therefore \ddot{\theta} + \frac{2}{3}k/m\,\theta = 0$$

$$\boxed{\omega_n = 2k/3m}$$

Q. Calculate the natural frequency of the spring mass pulley system given below.

Sol



Equation of motion

$$m\ddot{x}\, r + I\ddot{\theta} + (kx)\, r = 0$$

$$I = \frac{1}{2} M r^2, \quad x = r\theta \quad \ddot{x} = r\ddot{\theta}$$

$$m \cancel{r^2} \ddot{\theta} + \frac{1}{2} M \cancel{r^2} \ddot{\theta} + k \cancel{r^2} \theta = 0$$

$$\left(m + \frac{1}{2} M\right) \ddot{\theta} + k\theta = 0$$

$$\omega_n = \sqrt{\frac{k}{m + M/2}}$$

Using Energy method

KE Translation + KE Rot^n + PE = Constant.

$$= \frac{1}{2} m \dot{x}^2 + \frac{1}{2} I \dot{\theta}^2 + \frac{1}{2} k x^2 = \text{const}$$

$$= \frac{1}{\cancel{2}} m \cancel{r^2} \dot{\theta}^2 + \frac{1}{\cancel{4}\,2} M \cancel{r^2} \dot{\theta}^2 + \frac{1}{\cancel{2}} k \cancel{r^2} \theta^2 = \text{const}$$

Put $I = \frac{1}{2} M R^2$, $x = r\theta$

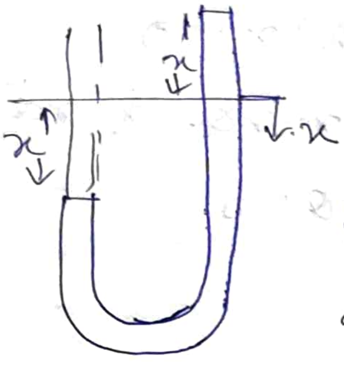
Diff. w.r.t time

$$m \cdot \cancel{2} \ddot{\theta} \cancel{\dot{\theta}} + \frac{M}{2} \cancel{2} \ddot{\theta} \cancel{\dot{\theta}} + k \cancel{2} \cancel{\dot{\theta}} \cdot \theta = 0$$

$$= \left(m + \frac{M}{2}\right) \ddot{\theta} + k\theta = 0$$

$$\omega_n = \sqrt{\frac{k}{m + \frac{M}{2}}} \text{ rad/sec.}$$

Q. A simple U tube manometer filled with liquid. Calculate the frequency of resulting if the minimum length of manometer tube is 'l'.



Let l = length of the U-tube contains fluid.
Here the liquid column is displaced from equilibrium position by a distance x. If ρ and A are the mass density of liquid and the crosssectional area of tube respectively. Mass of the liquid column is $\rho A l$ with the velocity $\dot{x}$, the $KE = \frac{1}{2}(\rho A l)\dot{x}^2$

$$PE = \rho(Axg)\cdot x = \rho A g x^2$$

Using energy method,

Total energy = KE + PE = const

$$\frac{1}{2}\rho A l \dot{x}^2 + \rho A g x^2 = \text{Const.}$$

Differentiating w.r.t time.

Using D-Alembert

$\rho A l \ddot{x} + (2xA\rho)g = 0$

$ma + \frac{\text{Restoring ..}}{2xw} = 0$

$= 2(\rho A x)g$

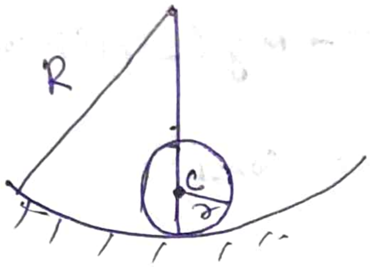
$$\frac{1}{2}\cancel{\rho}\cancel{A}l \times \cancel{2}\ddot{x}\cancel{\dot{x}} + 2\cancel{\rho}\cancel{A}g\cancel{\dot{x}}x = 0$$

$$l\ddot{x} + 2gx = 0$$

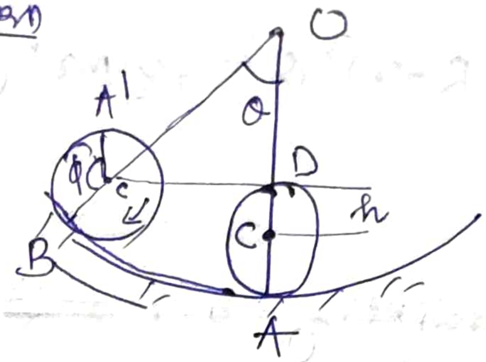
$$\ddot{x} + \frac{2g}{l}x = 0$$

$$\boxed{\omega_n = \sqrt{2g/l}}$$

Q A cylinder of mass M and radius r rolles without slipping on a circular surface of radius R. Find out the natural frequency of cylinder for small oscillation about the equilibrium point A, using energy method



FBD



When the cylinder rolles on a circular surface and takes the new position.

$$\text{Arc } A'B = \text{Arc } AB$$

$$= r\phi = R\theta \quad , \quad \text{or} \quad \phi = \frac{R\theta}{r}$$

Potential energy $= mgh$, where $h = CD = OC - OD$

$$= (R-r) - (R-r)\cos\theta$$

$$= (R-r)(1-\cos\theta)$$

$$\boxed{\text{so } PE = mg(R-r)(1-\cos\theta)} \quad \text{—— (1)}$$

KE - kinetic energy due to rotation of cylinder + due to translating motion.

As we know rotational (angular) displacement of cylinder

$$= (\phi - \theta)$$

so rotational velocity $= \dot{\phi} - \dot{\theta}$

$$KE_{\text{Rotational}} = \frac{1}{2} I (\dot{\phi} - \dot{\theta})^2 \quad , \text{ put } I = \frac{1}{2} m r^2$$

$$= \frac{1}{2}\left(\frac{1}{2} m r^2\right)\left(\frac{R\dot{\theta}}{r} - \dot{\theta}\right)^2 \qquad \phi = \frac{R\theta}{r}$$

$$\boxed{= \frac{1}{4} m r^2 \left(\frac{R}{r} - 1\right)^2 \dot{\theta}^2} \quad \text{—— (2)}$$

KE Translation

$$\frac{1}{2} m \dot{x}^2 = \frac{1}{2} m \left[(R-r)\dot{\theta}\right]^2$$

$$\boxed{= \frac{1}{2} m (R-r)^2 \dot{\theta}^2} \quad \text{—— (3)}$$

According to energy method, TE = constant.

$$(KE)_{\text{Trans}} + KE(\text{Rot}^n) + PE = \text{const.}$$

$$\frac{1}{2}M(R-r)^2\dot{\theta}^2 + \frac{1}{4}mr^2\left(\frac{R}{r}-1\right)^2\dot{\theta}^2 + mg(R-r)(1-\cos\theta)$$
$$= \text{const.}$$

~~Differentiating w.r.t time~~

or. $\frac{1}{2}M(R-r)^2\dot{\theta}^2 + \frac{1}{2}\cdot\frac{1}{2}mr^2\frac{(R-r)^2}{r^2}\dot{\theta}^2 + mg(R-r)(1-\cos\theta) = \text{const}$

or $\frac{3}{4}m(R-r)^2\dot{\theta}^2 + mg(R-r)(1-\cos\theta) = \text{const.}$

Differentiating w.r.t time we get.

$$\cancel{2}\cdot\frac{3}{\cancel{4}_2}\cancel{m}(R-r)^{\cancel{2}}\dot{\theta}\cdot\ddot{\theta} + \cancel{m}g\cancel{(R-r)}\sin\theta\cdot\dot{\theta} = 0$$

$$\frac{3}{2}(R-r)\ddot{\theta} + g\theta = 0 \qquad \text{(as } \sin\theta \simeq \theta\text{, when } \theta \text{ is very small)}$$

or $\boxed{\omega_n = \sqrt{\frac{2g}{3(R-r)}}\ \text{rad/sec}}$.

Q. A spring mass system has a natural frequency of 10 Hz. When the spring constant is reduced by 800 N/m the frequency is altered by 45%. Find the mass and spring constant of the original system.

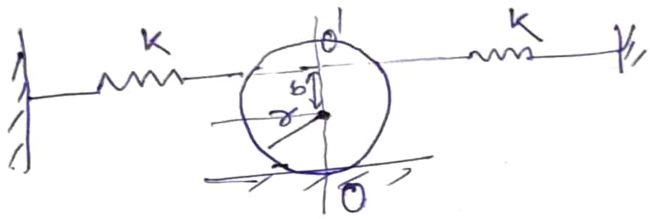
Soln $f = \frac{1}{2\pi}\sqrt{K/m}$, $10 = \frac{1}{2\pi}\sqrt{K/m}$, —— (1)

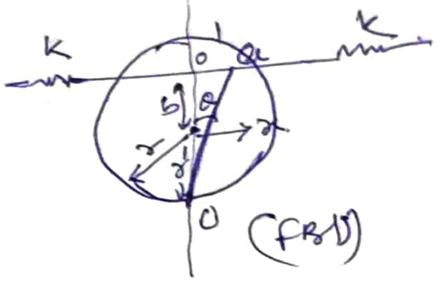
$$10\times 0.55 = \frac{1}{2\pi}\sqrt{\frac{K-800}{m}} \quad\text{—— (2)}$$

Solving these two ; $m = 0.2905$ kg

$K = 1146.98$ N/m

Q. find the natural frequency of the system as shown





(FBD)

Sol^n

$$KE = \frac{1}{2} M\dot{x}^2 + \frac{1}{2} I\dot{\theta}^2 \qquad (x = r\theta) \quad \dot{x} = r\dot{\theta} \qquad I = \frac{1}{2} mr^2$$

$$= \frac{1}{2} M r^2 \dot{\theta}^2 + \frac{1}{2} I \dot{\theta}^2$$

$$= \frac{1}{2} Mr^2 \dot{\theta}^2 + \frac{1}{4} mr^2 \dot{\theta}^2 = \frac{3}{4} mr^2 \dot{\theta}^2$$

from the FBD $\quad x = O'a = OO' \times \theta$, (as θ being small)

$$= (r+b)\,\theta$$

$$PE = \frac{1}{2}kx^2 + \frac{1}{2}kx^2 = 2\cdot\frac{1}{2}k[(r+b)\theta]^2 = k(r+b)^2\theta^2$$

$KE + PE =$ const.

$\frac{3}{4} mr^2 \dot{\theta}^2 + k(r+b)^2\theta^2 =$ const.

differentiating w.r.t time.

$$\frac{3}{4} mr^2 \ddot{\theta} + k(r+b)^2\theta = 0$$

$$\text{frequency} = \omega_n = \sqrt{\frac{k(r+b)^2}{\frac{3}{4} mr^2}} = \sqrt{\frac{4k(r+b)^2}{3mr^2}} \text{ rad/sec.}$$

Using D-alembert principle — Take moment abt. O (spring are $\parallel$)

$$(M\ddot{x})r + I\ddot{\theta} + 2k(r+b)((r+b)\theta) = 0 \qquad \text{Put } (x = r\theta)$$

$$\{M r^2\ddot{\theta} + \frac{1}{2}Mr^2\ddot{\theta}\} + 2k(r+b)^2\theta = 0$$

$$\frac{3}{2}mr^2\ddot{\theta} + 2k(r+b)^2\theta = 0$$

$$\ddot{\theta} + \frac{2k(r+b)^2}{\frac{3}{2}mr^2}\theta = 0 \longrightarrow \omega_n = \sqrt{\frac{4k(r+b)^2}{\frac{3}{2}mr^2}} \text{ rad/sec.}$$

Damped free vibration

Damping is the resistance offered by a body to the motion of vibratory system. The resistance may be applied by a liquid or solid internally or externally.

→ In this case the amplitude of the vibrating system decreases every cycle of motion. The ratio of decreasing depend upon the amount of damping.

→ The main advantage of providing damping in mechanical systems is to control the amplitude of the vibration so that the failure occurring because of resonance may be avoided

Note :- Resonance occurs when the frequency of excitation becomes equal to the natural frequency of the vibrating system.

Types of damping:-

There are mainly four types of damping used in mechanical system.

1) viscous damping
2) coloumb damping
3) structural damping
4) Non-linear, slip or Interfacial damping.

viscously damped free vibration (SDOF)

When the system is allowed to vibrate in a viscous medium, the damping is called as viscous.

→ In viscous damping, the damping force is proportional to the velocity of the body.

damping force $f \propto \dot{x}$

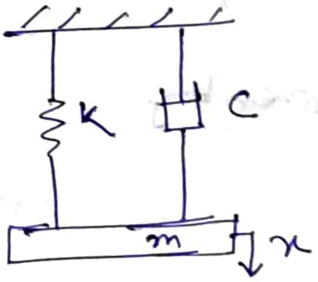
$$\boxed{\therefore f = c\dot{x}}$$

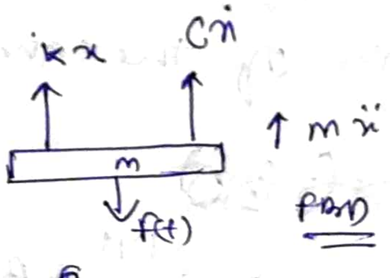
where c = damping coefficient.

Differential equation

Here a damped spring and mass system is the model for vibration analysis.

→ The damper is added ∥ to the spring to decrease amplitude of vibration in the spring mass system.





From the FBD, the equation of motion is seen

$$\boxed{m\ddot{x} + c\dot{x} + kx = f(t)} \longrightarrow (i)$$

The solution of this equation has two parts.

If $f(t) = 0$, we have the homogeneous differential equation whose sol^n corresponds physically to that free damped vibration.

→ If $f(t) \neq 0$, we obtain the particular solution that is due to the excitation irrespective of homogeneous sol^n.

with taking the homogeneous equation,

$$m\ddot{x} + c\dot{x} + kx = 0 \quad \text{—— (2)}$$

$$\ddot{x} + c/m\,\dot{x} + k/m\,x = 0 \quad \text{—— (3)}$$

The solution of the equation

$$D^2 + (c/m)\,D + k/m\,x = 0 \quad \text{—— (4)}$$

where $D = \frac{d}{dt}$, $D^2 = \frac{d^2}{dt^2}$

As the equation (4) is an ordinary differential equation of second order, the characteristic equation is

$$D^2 + (c/m)\,D + k/m\,x = 0 \quad \text{—— (5)}$$

The roots of the equations are

$$D_{(1,2)} = -(c/2m) \pm \sqrt{(c/2m)^2 - k/m} \quad \text{—— (4)}$$

The solution of the equation given by

$$x = Ae^{D_1 t} + Be^{D_2 t} \quad \text{—— (5)}$$

where A and B are the constants

The equation can be written as

$$x = Ae^{\left[-c/2m + \sqrt{(c/2m)^2 - k/m}\right]t} + Be^{\left[-c/2m - \sqrt{(c/2m)^2 - k/m}\right]}$$

critical damping coefficient (C_c)

The value of 'c' which makes the radical in equation (4) is zero called critical damping and denoted by C_c.

$$\left(\frac{C_c}{2m}\right)^2 - K/M = 0$$

or $$\left(\frac{C_c}{2m}\right)^2 = \frac{K}{M}$$

$$= \frac{C_c}{2m} = \sqrt{K/M} = \omega_n$$

or $$\boxed{C_c = 2m\omega_n} = 2m\sqrt{K/m} = 2\sqrt{\frac{Km^2}{m}} = \boxed{2\sqrt{Km}}$$

Damping ratio :- (ξ) – The ratio of damping coefficient to critical damping coefficient is termed as damping ratio.

$$\boxed{\xi = c/c_c} \Rightarrow \text{dimensionless}$$

Now $$c/2m = \frac{c}{c_c} \times \frac{c_c}{2m} = \xi\omega_n \quad [\because \text{Diving } N^r \text{ \& } D^r \text{ by } C_c]$$

or $$\boxed{c = 2\xi m\omega_n}$$

$$= 2\xi m\omega_n (K/K)$$

$$= 2\xi\omega_n \frac{K/K}{(M)}$$

$$= 2\xi \cancel{\omega_n} K/\omega_n^{\cancel{2}}$$

$$\boxed{C = 2\xi\frac{K}{\omega_n}}$$

As $D_{1,2} = -\left(\frac{c}{2m}\right) \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$

$= -w_n \xi \pm \sqrt{(w_n \xi)^2 - w_n^2}$

$$\boxed{D_{1,2} = w_n\left(-\xi \pm \sqrt{\xi^2 - 1}\right)}$$

Also the equation $\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0$ can be written as

$$\boxed{\ddot{x} + 2\xi w_n \dot{x} + w_n^2 x = 0}$$

The general solution becomes

$$x = A e^{\left(-\xi + \sqrt{\xi^2 - 1}\right) w_n t} + B e^{\left(-\xi - \sqrt{\xi^2 - 1}\right) w_n t}$$

Cases

The decision of oscillation behaviour depends upon the relation $\left(\frac{c}{2m}\right)^2 - \frac{k}{m}$

Case-1 If $\frac{c^2}{2m} > \frac{k}{m}$ or $\xi > 1$, overdamped system,

because the exponents are Real number.

→ Non-oscillatory

Case-2 $\frac{c}{2m} < \frac{k}{m}$, or $\xi < 1$, Under damped

→ Exponents are imaginary

→ oscillatory

Case-3 $\frac{c}{2m} = \frac{k}{m}$, critically damping

$\xi = 1$

→ Roots are real and equal

→ Non-oscillatory.

Undamped System ($\xi < 1$)

For the case of undendamped, the roots of equation, $\boxed{D(1,2) = \omega_n(-\xi \pm i\sqrt{1-\xi^2})}$, is beause the radical $\sqrt{\xi^2-1}$ is imagnary, where $i = \sqrt{-1}$

Let $\boxed{\omega_d = \omega_n \sqrt{1-\xi^2}}$, ω_d = damped natural frequeny

T_d = damped period $\frac{2\pi}{\omega_d}$

Now the solution becomes

$$x(t) = A e^{(-\xi + i\sqrt{1-\xi^2})\omega_n t} + B e^{(-\xi - i\sqrt{1-\xi^2})\omega_n t} \quad \text{—— (1)}$$

$$= e^{-\xi\omega_n t}(A e^{-i\omega_d t} + B e^{-i\omega_d t})$$

$$[\text{As } e^{\pm i\theta} = \cos\theta \pm i \sin\theta]$$

The above eqn (1) becomes

$$= e^{-\xi\omega_n t}(A\cos\omega_d t + A i \sin\omega_d t) + B\cos\omega_d t - Bi\sin\omega_d t)$$

$$= e^{-\xi\omega_n t}\{(A+B)\cos\omega_d t + i(A-B)\sin\omega_d t\}$$

$$\boxed{x(t) = e^{-\xi\omega_n t}(C_1 \cos\omega_d t + C_2 \sin\omega_d t)} \quad \text{—— (2)}$$

$$= e^{-\xi\omega_n t}(X\sin\phi\cos\omega_d t + X\cos\phi\sin\omega_d t]$$

where $C_1 = X\sin\phi$
$C_2 = X\cos\phi$

$$= e^{-\xi\omega_n t}[X\sin(\omega_d t + \phi)]$$

$$\boxed{x(t) = X e^{-\xi\omega_n t} \cdot \sin(\omega_d t + \phi)} \quad \text{—— (3)}$$

Note: For simplification or to remember the eqn (2) canbe written

$$\boxed{x(t) = e^{-\xi\omega_n t}(A\cos\omega_d t + B\sin\omega_d t)} \quad \text{—— (4)}$$

where A & B are constants can be found out be Initial condn.

Using the initial conditions

If $x(0) = x_0$ — (i)
$\dot{x}(0) = v_0$ — (ii)

Substituting in eqn (4).

$$x(t) = e^{-\xi\omega_n t}(A\sin\omega_d t + B\cos\omega_d t)$$

Using condn (i)

$$B = x_0$$

Using condn (ii)

$$\dot{x}(t) = e^{-\xi\omega_n t}\omega_d(A\cos\omega_d t - B\sin\omega_d t) - \xi e^{-\xi\omega_n t}[A\sin\omega_d t + B\cos\omega_d t]$$

putting $\dot{x}(0) = v_0$

$$= \omega_d A - \xi\omega_n B$$

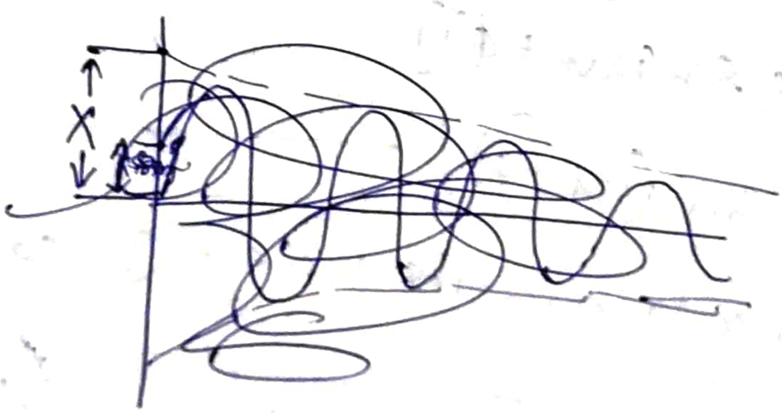
$$= \omega_d A - \xi\omega_n x_0$$

$$A = \frac{1}{\omega_d}[v_0 + \xi\omega_n x_0]$$

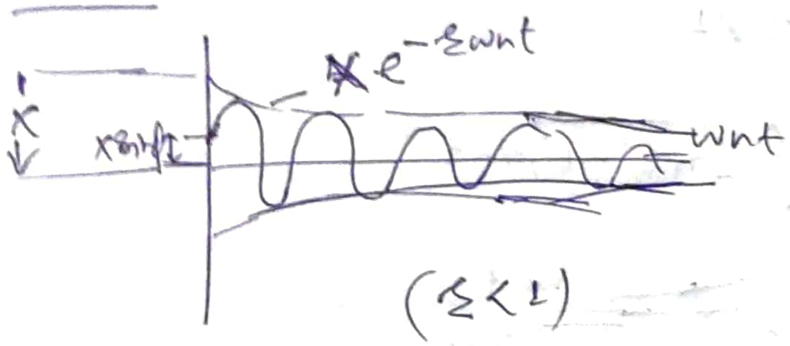
Now the solution becomes

$$x(t) = e^{-\xi\omega_n t}\left[\frac{1}{\omega_d}(v_0 + \xi\omega_n x_0)\sin\omega_d t + x_0\cos\omega_d t\right]$$

Response curve



Response curve



$(\xi < 1)$

Overdamped - Both the roots are real, distinct and positive. as $(\xi > 1)$.

i.e $D_{1,2} = (-\xi \pm \sqrt{\xi^2 - 1})\omega_n t < 0$

$$x(t) = e^{-\xi\omega_n t}\left(Ae^{(\sqrt{\xi^2-1}\,\omega_n)t} + Be^{(-\sqrt{\xi^2-1}\,\omega_n)t}\right) \quad \text{—(1)}$$

where A & B are the constants found out by initial conditions.

If $x(0) = x_0$ and $\dot{x}(0) = v_0$, then

using condⁿ (i) in above equation

$$\boxed{x_0 = A + B} \quad \text{——— (1)}$$

Now $\dot{x}(t) = e^{-\xi\omega_n t}\sqrt{\xi^2-1}\,\omega_n\left[Ae^{(\sqrt{\xi^2-1}\,\omega_n)t} - Be^{(-\sqrt{\xi^2-1}\,\omega_n)t}\right]$

$$-\xi\omega_n e^{-\xi\omega_n t}\left[Ae^{(\sqrt{\xi^2-1}\,\omega_n)t} + Be^{-(\sqrt{\xi^2-1}\,\omega_n)t}\right]$$

using condⁿ (ii)

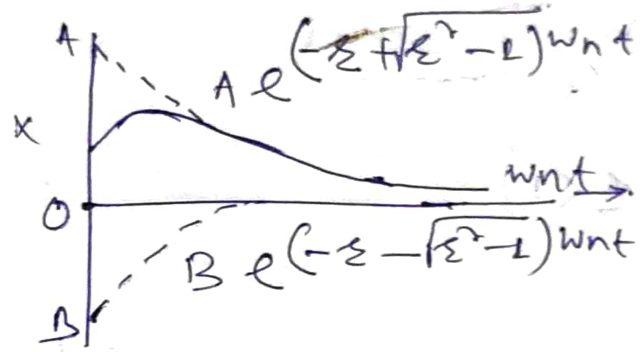
$$v_0 = \sqrt{\xi^2-1}\,\omega_n(A-B) - \xi\omega_n(A+B)$$

$$= \left(\sqrt{\xi^2-1} - \xi\right)\omega_n A - \left(\sqrt{\xi^2-1} - \xi\right)\omega_n B \quad \text{—— (2)}$$

On solving for A and B, we get

$$\boxed{\begin{aligned} A &= \frac{v_0 + x_0\left[\xi + \sqrt{\xi^2-1}\right]\omega_n t}{2\xi_n\sqrt{\xi^2-1}} \\ B &= \frac{-v_0 - x_0\left(\xi^2 - \sqrt{\xi^2-1}\right)\omega_n}{2\xi_n\sqrt{\xi^2-1}} \end{aligned}}$$

Response curve $\xi > 1$



(This is Aperiodic)

Critically damped System ($\xi = 1$) (Non oscillatory)

For critically damped system the roots are Real and equal

i.e. $D_1 = D_2 = \frac{-C_c}{2m} = -\omega_n$ $(\because C = C_c)$

The general solution becomes

$$\boxed{x = (A + Bt)e^{-\omega_n t}} \quad (1)$$ (Aperiodic or nonperiodic)

~~Again~~ substituting the initial condition

$x(0) = x_0$

$\dot{x}(t) = v_0$ in above equation

$A = x_0$ ——— (2)

Now $\dot{x}(t) = (A + Bt) - \omega_n e^{-\omega_n t} + Be^{-\omega_n t}$

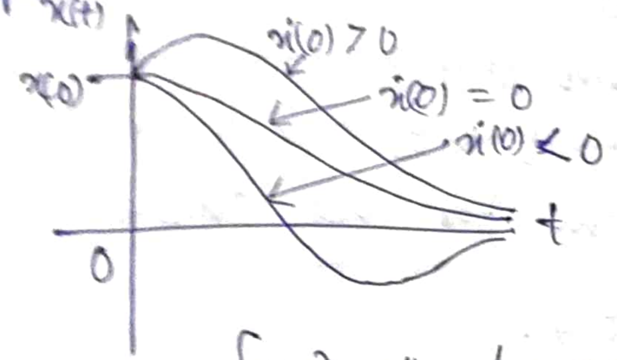
or $v_0 = -\omega_n A + B$

$$\boxed{v_0 = \omega_n . x_0 + B}$$

Hence the general solution becomes

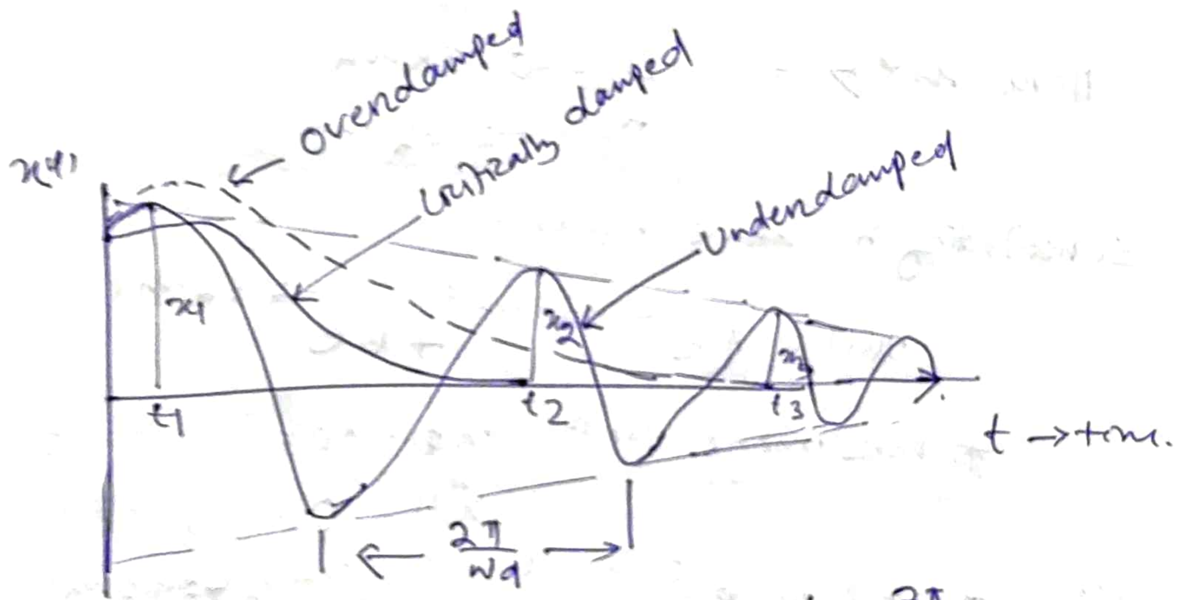
$$\boxed{x(t) = e^{-\xi\omega_n t}\left[x_0 + (v_0 + \omega_n x_0)t\right]}$$

Response curve



(critically damped) (Aperiodic)

All three types of damping response



$t_d = \frac{2\pi}{\omega_d}$

Q. The mass of a spring-mass-dashpot is displaced by a distance of 0.05m from the equilibrium position and released. Find the equation of the motion eq for the system for the case, when

ⓐ $\xi = 1.5$, ⓑ $\xi = 1$, ⓒ $\xi = 0.5$.

Ans
Case-I when $\xi = 1.5 > 1$, (System is over damped)

$$x(t) = Ae^{D_1 t} + Be^{D_2 t} \quad \text{—(1)} , \text{ where } D_{1,2} = (-\xi \pm \sqrt{\xi^2 - 1})\omega_n$$

Here for $\xi = 1.5$, $D_1 = -0.38\omega_n$
$D_2 = -2.62\omega_n$

Substituting in above equation.

$$x(t) = Ae^{-0.38\omega_n t} + Be^{-2.62\omega_n t} \quad \text{—(2)}$$

As per initial condn $x(t) = 0.05$, $t=0$
$\dot{x}(t) = 0$, $t=0$

$$\dot{x}(t) = -0.38A\omega_n e^{-0.38\omega_n t} - 2.62B\omega_n e^{-2.62\omega_n t} \quad \text{—(3)}$$

Using the above conditions

$$0.05 = A + B \quad \text{—(4)}$$

and $0 = -0.38A\omega_n - 2.62\omega_n B$

$$0 = 0.38A + 2.62B \quad \text{—(5)}$$

On solving eqn (4) & (5)

$A = 0.058$ $B = -0.008$

Then the solution

$$\boxed{x(t) = 0.058e^{-38\omega_n t} - 0.008e^{-2.62\omega_n t}}$$

case-2 when $\xi = 1$, critically damped.

$$x = (A + Bt)\, e^{-\omega_n t} \quad \text{--- (1)}$$

$$\dot{x}(t) = -\omega_n (A + Bt) e^{-\omega_n t} + B e^{-\omega_n t} \quad \text{--- (2)}$$

Applying bc's $x = 0.05$, $t = 0$

$\dot{x}(t) = 0$, $t = 0$

$$0.05 = A \quad \text{--- (3)}$$

and $0 = -\omega_n A + B$

or $B = \omega_n A = 0.05\,\omega_n$ --- (4)

Hence the solution becomes

$$x(t) = (0.05 + 0.05\,\omega_n t)\, e^{-\omega_n t}$$

$$\boxed{x(t) = 0.05(1 + \omega_n t)\, e^{-\omega_n t}}$$

case-3 when $\xi = 0.5$, underdamped ($\xi < 1$)

$$X(t) = X e^{-\xi \omega_n t} \sin(\sqrt{1-\xi^2}\,\omega_n t + \phi)$$

$$\dot{x}(t) = X e^{-\xi\omega_n t} \cdot (-\xi\omega_n) \cdot \sin(\sqrt{1-\xi^2}\,\omega_n t + \phi)$$

$$+ X\sqrt{1-\xi^2}\,\omega_n e^{-\xi\omega_n t} \cos(\sqrt{1-\xi^2}\,\omega_n t + \phi)$$

Again using Bc's,

$$0.05 = X \sin\phi$$

and $0 = X(-\xi\omega_n) \sin\phi + \sqrt{1-\xi^2}\,\omega_n X \cos\phi$

or $$\frac{\sin\phi}{\cos\phi} = \frac{\sqrt{1-\xi^2}\,\omega_n \cancel{X}}{\xi\omega_n \cancel{X}} = \frac{\sqrt{1-\xi^2}}{\xi} = 1.732.$$

$$\phi = \tan^{-1}(1.732) = 60°$$

and $X = \dfrac{0.05}{\sin\phi} = 0.058$

Hence the solution $x(t) = 0.058\, e^{-0.5\,\omega_n t} \sin(0.86\,\omega_n t + 60°)$

Q. A body of mass 6.5 kg is hung on a spring of spring stiffness 1.5 kN/m. It is pulled down 50 mm below its static equilibrium position and released. There is a frictional resistance which is proportional to the velocity and is 10 N when the velocity is 0.75 m/s. Calculate the time elapses and the distance which the body moves from the instant of release until it is again at rest at a height point of travel.

$\underline{\underline{Sol^n}}$ Given

$m = 6.5 kg$ $\quad K = 1.5 kN/m$ $\quad x(0) = 50 m$, $\quad F_d = 10 N$

$\dot{x} = 0.75 m/s$

As $F_d = c\dot{x}$, $\quad 10 = 0.75 c$, $\quad c = \frac{10 \times 4}{3} = \frac{40}{3} Ns/m$

$\omega_n = \sqrt{K/m} = \sqrt{\frac{1500}{6.5}} = 15.19$ rad/sec.

$C_c = 2m\omega_n = 2 \times 6.5 \times 15.19 = 197.48$ N-s/m.

$\xi = c/c_c = \frac{40}{3 \times 197.48} = \underline{0.0675} < 1$ (i.e. underdamped)

$\omega_d = \omega_n\sqrt{1-\xi^2} = 15.19\sqrt{1-(0.0675)^2} = 15.15$ rad/sec

$x(t) = e^{-\xi\omega_n t}[A \sin\omega_d t + B\cos\omega_d t)$

$\dot{x}(t) = e^{-\xi\omega_n t}\omega_d(A\cos\omega_d t - B\sin\omega_d t] - \xi\omega_n e^{-\xi\omega_n t}[A\sin\omega_d t + B\cos\omega_d t]$

Applying BCs $\quad x(0) = 50$
$\quad \dot{x}(0) = 0$

$B = 50 mm.$

$\dot{x}(t) = \dot{x}(0) = 0$, $\quad A = \frac{\xi\omega_n B}{\omega_d} = 3.384$

Time period $= T_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{15.15} = 0.4147$ sec

$$x(t) = e^{-0.0675 \times 15.19 \times 0.4147} \left[\cancel{3.84} \, 3.384 \sin(15.15 \times 0.4147) + 5\cos(15.15 \times 0.4147) \right]$$

$= 32.68$ mm

Q. A damper offers resistance 0.05 N at constant velocity 0.04 M/sec. The damper is used with $K = 9$ N/M. Determine the damping ~~and~~ ~~the~~ frequency of the system when the mass of the system is 0.10kg.

Soln $F_d = 0.05$ N, $\dot{x} = 0.04$ M/S, $K = 9$ N/M, $m = 0.10$ kg

As $F = c\dot{x}$

$$C = F/\dot{x} = \frac{0.05\,N}{0.04\,M/S} = 1.25\ N\text{-sec}/m$$

$$C_c = 2\sqrt{KM} = 2 \times \sqrt{9 \times .1} = 1.897\ N\text{-s}/M$$

$$\xi = C/C_c = \frac{1.25}{1.897} = 0.658 \text{ (underdamped)}$$

$$w_d = \sqrt{1-\xi^2}\, w_n = \sqrt{1-(0.658)^2} \times \sqrt{\frac{9}{0.10}} = 7.14 \text{ rad/sec}$$

Equation of motion of damped free vibration, when there is downward angular displacement 'θ' of ~~weight~~ mass m'

when the line displacement the eqⁿ is

$m\ddot{x} + c\dot{x} + kx = 0$, this is changes to

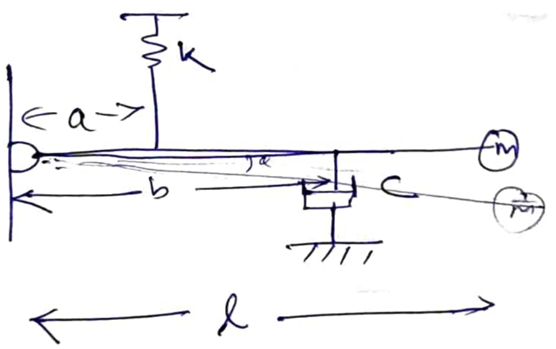
$I\ddot{\theta} + c_t\dot{\theta} + k_t\theta = 0$

also

$$\boxed{\begin{aligned} &\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 x = 0 \\ &\ddot{\theta} + 2\xi\omega_n\dot{\theta} + \omega_n^2\theta = 0 \end{aligned}}$$

Determine the natural frequency, damped frequency and critical damping of given fig.

Solⁿ



Equation of motion, when small angular displacement is given.

$$I\ddot{\theta} + (c \times b)\dot{\theta} \times b + (k a\theta)\cdot a = 0$$

putting $I = ml^2$,

$ml^2\ddot{\theta} + cb^2\dot{\theta} + ka^2\theta = 0$; $\ddot{\theta} + \frac{cb^2}{ml^2}\dot{\theta} + \frac{ka^2}{ml^2}\theta = 0$

$\ddot{\theta} + 2\xi\omega_n\dot{\theta} + \omega_n^2\theta = 0$

$c_t = cb^2$, $k_t = ka^2$

$\omega_n = \sqrt{\frac{ka^2}{ml^2}} = \frac{a}{l}\sqrt{k/m}$ rad/sec.

Critical damping coefficient $C_c = 2I\omega_n = 2al\sqrt{km}$

$\xi = c/c_c = \frac{cb^2}{2al\sqrt{km}}$

Damped natural frequency $= \omega_d = \omega_n\sqrt{1-\xi^2} = \frac{1}{2l^2m}\sqrt{4a^2l^2km - c^2b^4}$

Logarithmic decrement

It represents the ratio of decay of a free vibration.

It is defined as the natural logarithm of the ratio of any two successive amplitude.

The general solution of free damped vibration is $x(t) = X e^{-\xi \omega_n t} \sin(\omega_d t + \phi)$

$$\delta = \ln\left(\frac{x_1}{x_2}\right)$$

$$= \ln\left[\frac{e^{-\xi\omega_n t_1}(\sin \omega_d t + \phi)}{e^{-\xi\omega_n(t_1+T_d)}[\sin \xi\omega_d(t_1+t_d)+\phi]}\right]$$

Since the value of sine are equal when the time is increased by damped period T_d, then the above relation reduces to

$$\delta = \ln\left[\frac{e^{-\xi\omega_n t_1}}{e^{-\xi\omega_n(T_d+t_1)}}\right]$$

$$\boxed{\delta = \xi\omega_n T_d}$$

Now $T_d = \dfrac{2\pi}{\omega_d} = \dfrac{2\pi}{\omega_n\sqrt{1-\xi^2}}$

Hence $\boxed{\delta = \dfrac{2\pi\xi}{\sqrt{1-\xi^2}}}$

When $\xi \ll 1$ then

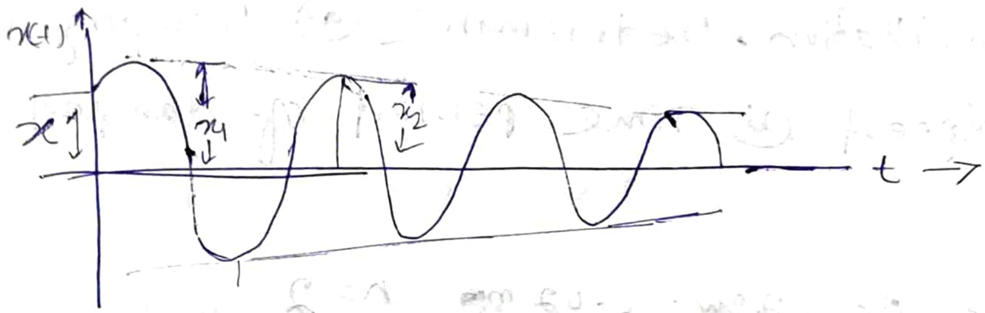
$$\boxed{\delta = 2\pi\xi}$$

Let the initial amplitude be x_0 & x_n be the amp at nth cycle

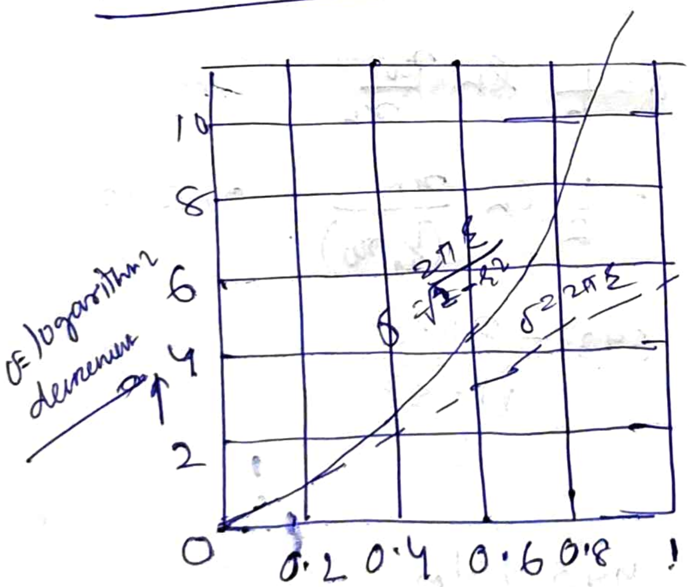
Now $\delta = \ln \frac{x_0}{x_1} = \ln \frac{x_1}{x_2} = \cdots + \ln \frac{x_{n-1}}{x_n}$

Also $\frac{x_0}{x_n} = \frac{x_0}{x_1} \times \frac{x_1}{x_2} \times \cdots \times \frac{x_{n-1}}{x_n} = e^{n\delta}$

$$\boxed{\delta = \frac{1}{n} \ln \frac{x_0}{x_n}}$$



Logarithmic decrement as function of ε



$\varepsilon = \frac{c}{c_c}$ ⟶ damping factor

Q. A mass of 10 kg is supported on spring which deflect by 2cm under the dead weight of mass. The vibration of the system are constrained to be linear and vertical and are damped by a dashpot which reduces the amplitude to one quarter of its original value in two complete oscillation. Determine (a) Damping force at unit speed (b) Time period of damped vibn.

Soln

$m = 10 \text{ kg}$ $\quad \delta_{st} = 2\text{cm} = 0.02 \text{ m}$ $\quad n = 2.$

$$\omega_n = \sqrt{g/\delta_{st}} = \sqrt{\frac{9.81}{0.02}} = 22.15 \text{ rad/sec}$$

Logarithmic decrement $= \delta = \frac{1}{n} \ln \frac{x_0}{x_n}$

$$= \frac{1}{2} \ln \frac{x_0}{(\frac{1}{4} x_0)} = 0.693$$

Also $\quad \delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$, $\quad 0.693 = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$

$\xi = 0.11$

$C_c = 2m\omega_n = 2 \times 10 \times 22.15 = 443 \text{ Ns/m}.$

(a) Damping force at unit velocity. C

$C = \xi C_c = 0.11 \times 443 = 48.73 \text{ Ns/m}$

(b) Time period T_d.

$\omega_d = (\sqrt{1-\xi^2})\omega_n = \sqrt{1-(0.11)^2} \times 22.15$

$= 22 \text{ rad/sec.}$

$$T_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{22} = 0.285 \text{ sec}$$

Q·2 - A body of mass 5kg is supported on a spring of stiffness 1000 N/M and has a dashpot connected to it which produces a resistance of 0·01 N at velocity of 1 M/s. In what ratio will be amplitude of vibration can be reduced after 5 cycle.

Soln $m = 5kg$ $K = 1000\ N/M$ $f_d = 0.01 N$ $\dot{x} = 1 M/s$

$m = 5$

As $F_d = c\dot{x}$, $0.01 = C \times 1$, $C = 0.01\ N S/m$.

$C_c = 2 m \omega_n = 2\sqrt{KM} = 2 \times \sqrt{1000 \times 5} = 141.4\ N\text{-}s/m$

Damping ratio $\xi = C/C_c = \frac{0.01}{141.4} = 7.07 \times 10^{-5}$

Logarithmic decrement $\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}} = \frac{2\pi \times 7.707 \times 10^{-5}}{\sqrt{1-(7.707 \times 10^{-5})^2}}$

$= 0.44 \cdot 10^{-3}$

also $\delta = \frac{1}{n} \ln \frac{x_0}{x_n}$

$0.44 \times 10^{-3} = \frac{1}{5} \ln \frac{x_0}{x_n}$, $\boxed{\frac{x_0}{x_n} = 1.002}$

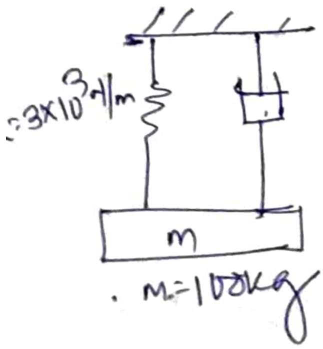
Q-3 If the damping provided is only 25% of the critical value find out 1) Damping factor

2) critical damping coefficient

3) damped natural frequency

4) Logarithmic decrement

5) Ratio of consecutive amplitude



Sol^n Given $m = 100\,kg$ $\quad k = 30 \times 10^3\,N/m$ $\quad C = 0.25\,C_c$

1) $\xi = C/C_c = 0.25$

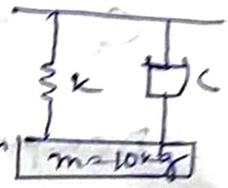
ii) $C_c = 2\sqrt{KM} = 3464.1\,NS/M$

iii) $\omega_d = \sqrt{1-\xi^2}\,\omega_n = \sqrt{1-(0.25)^2}\sqrt{K/M} = 16.77\,rad/sec.$

iv) $\delta = \dfrac{2\pi\xi}{\sqrt{1-\xi^2}} = \dfrac{2\pi \times 0.25}{\sqrt{1-(0.25)^2}} = 1.622$

v) $\dfrac{x_0}{x_n} = e^{\delta} = e^{1.622} = 5.063$

Q The mass of 10 kg makes 40 oscillations in 20 sec. The amplitude decreases to 0.20 of the initial value after 5 oscillations. Find out i) Stiffness K, ii) Logarithmic decrement (δ) iii) damping const (C)



Sol^n Given $m = 10\,kg$ $\quad N = 40$ $\quad t = 20\,sec$

The natural frequency $f = \dfrac{N}{T} = \dfrac{40}{20} = 2\,Hz$

As $\omega_n = \sqrt{K/M}$

$2\pi f_n = \sqrt{K/M}$

$\Rightarrow 2\pi\, 2 = \sqrt{K/10}$ $\Rightarrow \boxed{K = 1579.14\,N/m}$

(ii)

$\dfrac{x_0}{x_5} = \dfrac{x_0}{x_1} \times \dfrac{x_1}{x_2} \times \cdots \times \dfrac{x_4}{x_5}$

As $\dfrac{x_0}{x_1} = \dfrac{x_1}{x_2} = \cdots = \dfrac{x_4}{x_5}$

$\dfrac{x_0}{x_5} = \left(\dfrac{x_0}{x_1}\right)^5 \Rightarrow \dfrac{x_0}{0.2\,x_0} = \dfrac{x_0}{x_1}$, or $\dfrac{x_0}{x_1} = 1.38$

So $\delta = \ln \frac{x_0}{x_1} = \ln 1.38 = 0.322$

iii)

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

$$0.322 = \frac{2\pi\xi}{\sqrt{1-\xi^2}} \quad . \quad \xi = 0.051$$

iv) $\xi = c/c_c = \frac{c}{2mw_n}$ or $c = 2m\xi w_n = 12.82\,Ns/m$

Q - A body of mass 1.25 kg is suspended from a spring with a scale of 2kN/m. A dashpot attached between the mass and the ground and has a resistance of 0.5 N at a velocity 50mm/sec.

Determine - a) Natural frequency of the system. $\to w_n$

(b) critical damping factor $\to \xi$

c) Ratio of successive amplitude $\to \frac{x_0}{x_1}$

d) Amplitude of the body 10 cycles after it was released from the initial displacement of 20mm $\to x_n$

(e) The displacement of the body exactly 1.25 sec after it was released from the initial displacement of 20mm. $\to x(t)$

Given

$M = 1.25\,kg \quad K = 2\,kN/m \quad F_d = c\dot{x} = 0.5\,N \quad \dot{x} = 50\,mm/sec$
$= 50 \times 10^{-3}\,m/s$

(a) $\omega_n = \sqrt{k/m} = \sqrt{\frac{2000}{1.25}} = 40 \text{ rad/sec}$

(b) $C_c = 2m\omega_n = 2 \times 1.25 \times 40 = 100 \text{ NS/m}$

(c) $\xi = c/c_c = \frac{10}{100} = 0.1$

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}} = \frac{2\pi \times 0.1}{\sqrt{1-(0.1)^2}} = 0.6315$$

$$\delta = \ln\left(\frac{x_0}{x_1}\right)$$

$$\frac{x_0}{x_1} = e^{\delta} = e^{0.6315} = 1.88$$

(d) as $n = 10$ $\quad \delta = \frac{1}{n} \ln \frac{x_0}{x_n}$

$$0.6315 = \frac{1}{10} \ln\left[\frac{20}{x_n}\right]$$

$$x_n = 0.36$$

(e) $\omega_d = \omega_n\sqrt{1-\xi^2} = 39.8 \text{ rad/sec}$

$$x(t) = e^{-\xi\omega_n t}(A \sin\omega_d t + B\cos\omega_d t)$$

$x(0) = 20$, $B = 20$

$$\dot{x}(t) = e^{-\xi\omega_n t}\omega_d(A\cos\omega_d t - B\sin\omega_d t) - \xi\omega_n e^{-\xi\omega_n t}\left[A\sin\omega_d t + B\cos\omega_d t\right]$$

$\dot{x}(0) = 0$

$$\dot{x}(0) = A\omega_d - \xi\omega_n B$$

$$0 = 29.8A - 0.1 \times 40B \quad , \; A = 2.01$$

$x(1.25) = 0.1106 \text{ mm}$

Q. The following data are given for a vibrating system with viscous damping. $m = 4.5 kg$, $k = 5250 N/m^2$, and $c = 21 NS/m$. Determine the logarithmic decrement and ratio of any two successive amplitude.

Soln Given $m = 4.5 kg$, $k = 5250 N/m$, $c = 21 NS/m$

Undamped natural frequency of the system in Radian per second is

$$W_n = \sqrt{K/m} = \sqrt{\frac{5250}{4.5}} = 34.2 \text{ rad/sec}$$

The critical damping coefficient $C_c = 2 m W_n$

$$= 2 \times 4.5 \times 34.2$$

$$= 307.4 \text{ NS/m}$$

$$\xi = \frac{c}{C_c} = \frac{21}{307.4} = .0683$$

Logarithmic decrement

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}} = \frac{2\pi \times 0.0683}{\sqrt{1-(0.0683)^2}} = 0.430$$

Two successive amplitude.

$$\frac{x_1}{x_2} = e^{\delta} = e^{0.430} = 1.54$$

Q. Show that the logarithmic decrement is also given by equation $\delta = \frac{1}{n} \ln \frac{x_0}{x_n}$, where x_n represents the amplitude after n cycle have elapsed. Plot a curve giving the number of cycle against ξ for the amplitude diminish by 50 percent.

Sol^n As $\delta = \ln \frac{x_0}{x_1} = \ln \frac{x_1}{x_2} = \cdots = \ln \frac{x_{n-1}}{x_n}$

Also $\frac{x_0}{x_n} = \left(\frac{x_0}{x_1}\right)\left(\frac{x_1}{x_2}\right) \cdots = e^{n\delta}$

or $\delta = \frac{1}{n} \ln \frac{x_0}{x_n}$

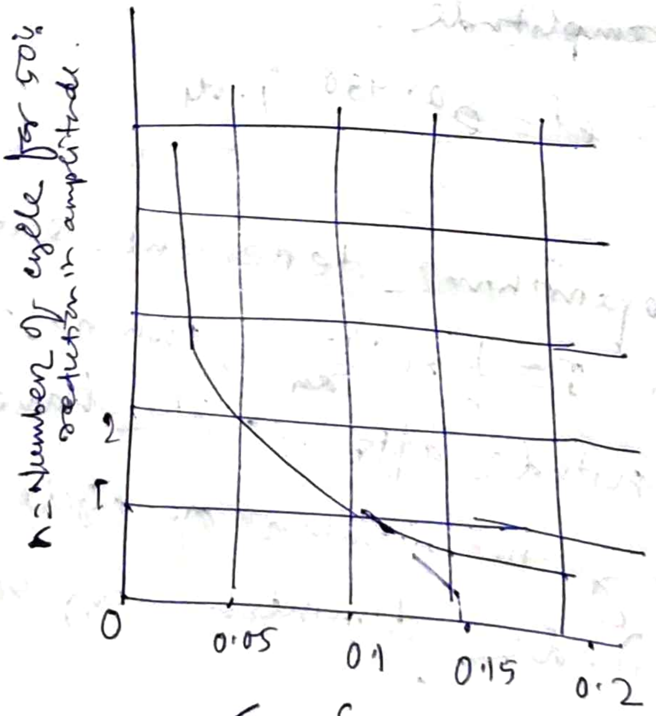
To determine the number of cycles elapsed for a 50% red^n in amplitude, we obtained the following relation from preceding eq^n.

$$\delta \cong 2\pi\xi = \frac{1}{n} \ln \frac{x_0}{(x_0/2)} = \frac{1}{n}\ln 2$$

$$= \frac{0.693}{n}$$

$$n\xi = \frac{0.693}{2\pi} = 0.110$$

This equation stands for hyperbola..



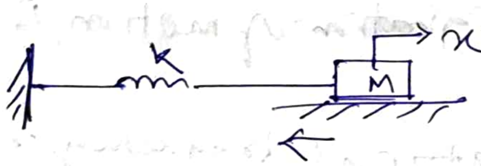
$\xi = c/c_c$ = damping factor.

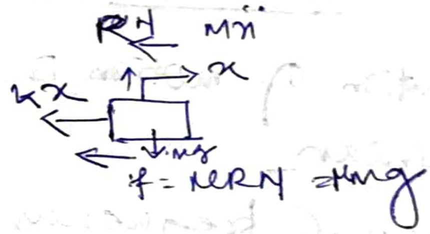
Coulomb damping

Friction between two dry surfaces caused dry friction which is nothing but coulomb damping whenever the components of structure slides relative to each other dry friction damping appears internally.

The damping force is equal to the product of normal force and the coefficient of friction μ and assumed to be independent of velocity, once motion is initiated.

Case-1 If mass is moving to right :





$$M\ddot{x} + kx + f = 0$$

$$\ddot{x} + \frac{k}{m}x + f/m = 0$$

$$\ddot{x} + k/m\left[x + f/k\right] = 0 \quad \text{—— (1)}$$

Let $y = x + f/k$

$$\ddot{y} = \ddot{x}$$

$$\ddot{y} + k/m\, y = 0 \quad \text{—— (2)}$$

Then the general soln of the eqn.

$$y(t) = A \sin w_n t + B \cos w_n t$$

$$x(t) = A \sin w_n t + B \cos w_n t - f/k$$

The system vibrates about $x = -f/k$

case-2 If mass moves right to left.

In this case when the body starts moving towards left after being displaced towards right.

The eqn of motion

$$m\ddot{x} + kx - f = 0$$

Solution to this equation.

$$x(t) = A \sin \omega_n t + B \cos \omega_n t + f/k$$

The System vibrates about $x(t) = f/k$

Difference between coulomb and viscous damp

Coulomb damping	viscous damping
① Equation of motion is nonlinear	① Equation of motion is linear
2. Natural frequency unaltered with addition of coulomb damping	2. Natural frequency reduced with addition of viscous damp
3. periodic motion	3. Non periodic (overdamped)
4. System comes to rest after some time	4. Motion theoretically continuous for ever
5. Amplitude reduced linearly	5. Amplitude reduced exponentially.

Q. A horizontal spring mass system with coulomb damping has a mass of 5.0 kg attached to a spring of stiffness 980 N/m. If the coefficient of friction is 0.025, calculate

ⓐ The frequency of free oscillations.

ⓑ The number of cycles corresponding to 50% reduction in amplitude if initial amplitude is 5.0 cm; and.

ⓒ the time taken to achieve this 50% reduction.

Sol^n $m = 5kg$, $k = 980 N/m$ $\mu = 0.025$

Sol^n ⓐ Natural frequency $\omega_n = \sqrt{k/m} = \sqrt{\frac{980}{5}} = 14$ rad/sec.

$f_n = \frac{\omega_n}{2\pi} = \frac{14}{2\pi} = 2.23 Hz$.

ⓑ Amplitude after 50% reduction is half of the initial amplitude, i.e. 0.025 m.

As ~~~~ Reduction in amplitude per cycle

$F = \mu mg = 0.025 \times 5 \times 9.81 = 1.226 N$ $= 4F/k = \frac{4 \times 1.226}{980} = 5 \times 10^{-3} m$.

cycles completed in 50% reduction.

$$= \frac{0.025 \ m}{0.005 \ m} = 5 \text{ cycles.}$$

ⓒ Time taken to achieve 50% reduction.

$= \text{No. of cycles} \times \frac{2\pi}{\omega_n} = 5 \times \frac{2\pi}{14} = 2.24$ sec

Amplitude decay in coulomb damping

$$x(t) = A\sin\omega_n t + B\cos\omega_n t + F/k \quad \text{--- (1)}$$

~~This solution holds good for half the cycle~~

~~when $t = \pi/\omega_n$, half cycle is complete.~~

→ for solving taking initial conditions.

$$x = x_0, \quad t = 0$$

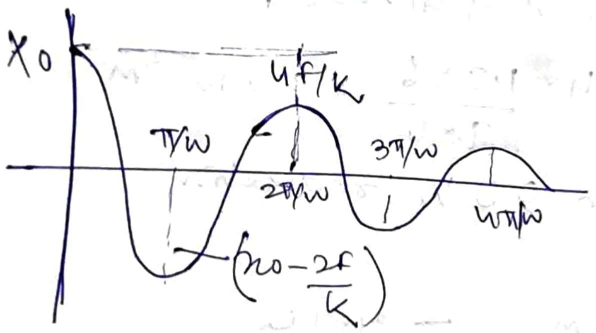
$$\dot{x}(0) = v_0 \quad t = 0$$

Using these two BC's

$$B = (x_0 - F/k), \quad D = 0$$

Substituting in above equation.

$$x = (x_0 - F/k)\cos\omega_n t + F/k \quad \text{--- (2)}$$



The above eqn (2) holds good for half the cycle, when, $t = \pi/\omega_n$, half the cycle is complete. So displacement for half the cycle can be obtained from the above eqn

$$x = (x_0 - F/k)\cos\pi + F/k$$

$$= -(x_0 - F/k) + F/k = -(x_0 - 2F/k)$$

This is the amplitude for left extreme position of the body. It is clear that the initial displacement x_0 is reduced by $2f/k$.

→ In next half cycle when the body moves to the right the initial displacement will be reduced by $2f/k$.

→ for in one complete cycle the amplitude reduced by $4F/k$.

Q-1 - A mass of 1 kg is attached to a spring having a stiffness of 3920 N/m. The mass slides on a horizontal surface, the coefficient of friction between mass and surface ~~beg~~ being 0.1. Determine the frequency of vibration of the system and amplitude of after one cycle if the initial amplitude is 0.25cm.

Soln

$m = 1\,kg \quad k = 3920 \quad \mu = 0.1$

$\omega_n = \sqrt{k/m} = \sqrt{\frac{3920}{1}} = 62.6\ rad/sec.$

$Force = \mu mg = 0.1 \times 1 \times 9.8 = 0.98\ N$

$Reduction\ in\ amplitude/cycle = 4f/k = \frac{4 \times 0.98}{3920}$

$= 0.001\ m.$

Final amplitude

$= 0.0025 - 001 = 0.0015\ m$

Q A vertical spring mass system has a mass of 0.5kg and an initial deflection 0.2cm. The system is subjected to coulomb damping when displaced by 2cm from the equilibrium position and released, it under goes complete 10 cycles and comes to rest in the extreme position on the side which it was displaced. Determine the final rest position.

Soln $m = 0.5kg$ $\delta_{st} = .2cm = .2 \times 10^{-2} m$ $x_0 = 2cm = 2\times10^{-2} m.$

$n = 10$

$\omega_n = \sqrt{k/m} = \sqrt{g/\delta_{st}}$ $\Rightarrow k/m = g/\delta_s$

$k = \frac{mg}{\delta_{st}} = \frac{0.5 \times 9.81}{0.2\times10^{-2}} = 2448.023 N/m$

No. of cycles $= \frac{\text{Total amplitude reduction}}{\text{Amplitude reduction/cycle}}$

$10 = \frac{2\times10^{-2}}{4F/k} = \frac{2\times10^{-2}}{4F/2448.023 N/m}$

$F = 1.224N$

As $F = \mu mg$, $1.224N = \mu \times 0.5 \times 9.81$

$\mu = 0.25$

$$x = (x_0 - f_0/k)\cos\sqrt{k/m} + f/k$$

$$= \left(2\times10^{-2} - \frac{1.223}{2,448.023}\right)\cos\sqrt{\frac{2448.023}{0.5}}\,t + \frac{1.224}{2448.023}$$

$$t = 10\times\frac{1}{f_n} = 10\times\frac{1}{2\pi}\sqrt{g/\delta_{st}} = 10\times\frac{1}{2\pi}\sqrt{\frac{9.81}{0.2\times10^{-2}}} =$$

$$= 0.8975 \text{ sec}$$

$$x = \left((2\times10^{-2}) - \frac{1.223}{2.488.023}\right)\cos(6279) + \frac{1.224}{2448.023}$$

$$= 0.01998 \text{ m}$$

Energy dissipated by damping

The energy is either dissipated into heat or radiated away.. In steady state forced vibration the loss of energy is balanced by the energy which is supplied by excitation. The energy lost/cycle due to damping force f_d is computed from the equation.

$$W_d = \oint f_d\,dx$$, where W_d = depends upon the factors such as temperature, frequency or amplitude.

As we know damping force

$$F_d = c\dot{x} \quad \text{——} (1)$$

with the steady state displacement and velocity

$$x = X\sin(\omega t - \phi)$$

$$\dot{x} = \omega X\cos(\omega t - \phi) \quad \text{——} (2)$$

Energy dissipated / cycle

$$W_d = \oint c\dot{x}\,dx = \oint c\dot{x}^2 dt \quad , \text{ as } \frac{dx}{dt} = \dot{x}, \quad dx = \dot{x}\,dt$$

$$= c\omega^2 X^2 \int_0^{2\pi/\omega} \cos^2(\omega t - \phi)\,dt$$

$$\boxed{W_d = \pi c \omega X^2}$$ $(\because 2\pi/\omega = t$ for full cycle$)$

At resonance, as $\omega = \omega_n$ and $c = 2\xi\sqrt{KM}$

$$\boxed{W_d = \pi\, 2\xi K X^2}$$

Equivalent viscous damping: (C_{eq})

The equivalent viscous damping C_{eq} is found by equating the energy dissipated by viscous damping to that of non-viscous force with assumed harmonic motion.

The Energy dissipated in equivalent viscous damping is

$$\boxed{W_d = \pi C_{eq} \omega X^2}$$

~~Here~~ where W_d, must be evaluated for the particular types of damping.

Structural damping

When the materials are cyclically stressed, energy dissipated internally with in the material itself. Some materials like steel, Aluminium, the energy dissipated/cycle is independent of frequency over a wide frequency range and proportional to the square of the amplitude of vibration. Internal damping fitting, this classification called solid damping or structural damping

The energy dissipated by structural damping may be written as

$$W_d = \alpha X^2$$

where α is the constant with unit force/displacement.

From equivalent viscous damping

$$\pi C_{eq} \omega \cancel{X^2} = \alpha \cancel{X^2}$$

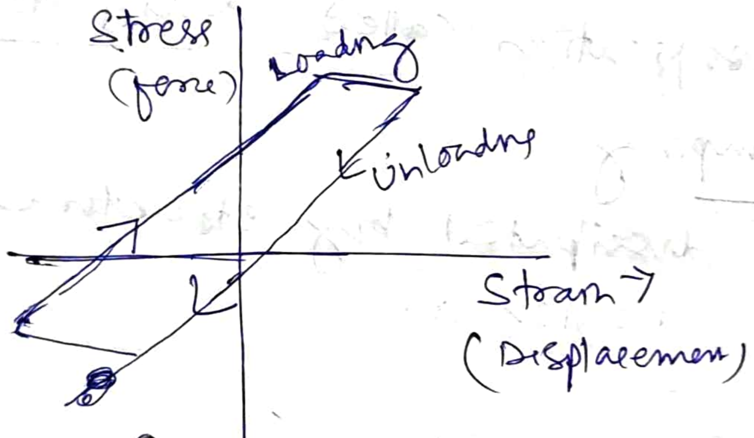
$$\boxed{C_{eq} = \frac{\alpha}{\pi \omega}}$$

Substituting of C_{eq} for C the differential equation of motion for a system with structural damping may be written as

$$\boxed{m\ddot{x} + \left(\frac{\alpha}{\pi \omega}\right)\dot{x} + kx = f(t)}$$

In structural damping the magnitude of damping is very small as compared to other damping.

→ Experimentally shows that for elastic materials for loading and unloading condition a loop is formed on stress strain curve the loop is called ~~its~~ hysteresis loop.



(Hysteresis loop)

The area of this loop is the amount of energy dissipated in one cycle during vibration. This type of loop is known as hysteresis loop.

Q. Determine the power required to vibrate a spring mass system with an amplitude of 15 cm. and at a frequency of 100 Hz. The system has a damping factor 0.05 and a damped natural frequency of 22 Hz as found out from the vibration record. The mass of the system is 0.5 kg

Soln $X = 0.15\,m$, $f = 100\,Hz$, $\xi = 0.05$, $m = 0.5$

$f_n = 22\,Hz$ $\omega_n = 2\pi \times 22 = 44\times\pi$ rad/sec

$C_{eq} = 2 m \omega_n \xi = 2 \times 0.5 \times 44 \times \pi \times 0.05$

$= 6.90$

$\omega = 100 \times 2\pi$ rad/sec $= 200\pi$ rad/sec.

Energy dissipated per cycle

$W_d = \pi\, C_{eq}\, \omega\, X^2$

$= \pi \times 6.90 \times 200\pi \times (0.15)^2$

$= 306.5$ N-m